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UTILITIES, PSYCHOLOGICAL VALUES, AND THE TRAINING OF DECISION-MAKERS

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SUMMARY

M. Allais and the "American School" have formulated two alternative hypotheses for the decision-making under uncertainty. Although ~~Allais~~ concentrates on the case when the consequences are money-amounts, his hypothesis can be applied to more general sets of consequences, as used in the "American" discussion. In both hypotheses, subjective probabilities can be revealed by similar observational methods, but Allais' 'psychological values' and the 'Bernoullian utilities' are, in general, not identical concepts. In particular, Allais' maximand depends, not only on the mean but also on the dispersion of "psychological values" -- an assumption that Allais considers necessary to explain attitudes towards the variance of money amounts.

Neither hypothesis can claim to describe the 'actual' behavior of un-trained, unexperienced, non-reflecting members of our or other cultures. Rather, these, or some other, weaker hypotheses claim to be models of 'rational' behavior. A challenge to psychologists: develop methods to train rational decision-makers!

Letter on file

A

I. DESCRIBING OR PRESCRIBING? [1)]

On a May day in 1952, between sessions of the International Colloquium on Risk, Professor Allais drove his luncheon guests to the Jockey Club on the outskirts of Paris. The guests, -- Ragnar Frisch, L.J. Savage and myself -- alighted from his car. Maurice Allais manoeuvred to park it and got out. In another car, the driver stepped loudly on his brake: crossing the road in a hurry, Allais has just avoided to be hit. Instantly, Savage snapped his fingers "Bob Thrall should be here!" For our host's actions contradicted the lexicographical preference ordering that Professor Thrall had defended in the morning session and that would, of course, rank "surviving but letting one's guests wait one more minute" ahead of the smallest chance of being run over.

Savage was thus arguing from observed behavior. Yet his Foundations of Statistics published two years later profess to be mainly normative, prescriptive, -- not descriptive.[2)] Yet the boundary is not easy to trace. Before reporting (Foundations, 5.6) on his own reactions to Allais brilliant, deeply searching experiment (see now Allais: 63) Savage reminds us that D. Bernoulli was indeed led by observed behavioral facts (the Petersburg paradox) to replace mathematical by moral expectation. When subjected to Allais' experiment Savage "immediately expressed preference" in a way contradicting his own 'sure-thing principle'. But, by tabulating two payoff matrices, with drawn numbered lottery

tickets as events, and two pairs of alternative prospects as acts, the problem became transparent. He must revise the choice he made either between the members of the first, or of the second pair of acts. He has thus

> "corrected an error. There is, of course, an important sense in which preferences, being entirely subjective, cannot be in error; but in a different, more subtle sense they can be. Let me illustrate by a simple example containing no reference to uncertainty. A man buying a car for $2,134.56 is tempted to order it with a radio installed, which will bring the toatl price to $2,228.41, feeling that the difference is trifling. But, when he reflects that, if he already had the car, he certainly would not spend $93.85 for a radio for it, he realizes that he has made an error."

I would add: "and he regrets the loss caused by his inconsistency". For this may be the crucial test. It is instructive to read how it solved some of B. de Finetti's hesitations (1972, pp. 150ff) about the nature of subjectivistic probability theory.

Another example of 'reflection and regret': in essence, Pratt, Raiffa and Schlaifer (1965, Section 2.3.4) convince you that you would regret your 'intransitive' ('cyclical') behavior, as follows: "You own <u>A</u> and prefer <u>A</u> to <u>B</u>, <u>B</u> to <u>C</u>, and (cyclically!) <u>C</u> to <u>A</u>. Therefore, give us <u>A</u> and pay us a

premium, to obtain C. Now pay a premium to exchange C for B. Finally, pay a premium to get A in exchange for B. You end up with your original possession of A, minus three premiums... Don't you now regret having acted as you did?"

We are reminded of Socrates' dialogues, teaching wisdom by facing you with the consequences of your folly. And again: the distinction between prescriptive logic (and mathematics, and formal ethics if it exists) and descriptive psychology is a subtle one. We first proceed to contrasting two psychological hypotheses.

II. TWO HYPOTHESES AND SOME COMMENTS

II.1 It seems to me that, as a matter of descriptive psychology, the two hypotheses contrasted by M. Allais -- his own and that of the "American School" can be stated as on Figure 1.

Figure 1

TWO HYPOTHESES

Allais (13-24: "Fundamental Factors)	Ramsey(1926),De Finetti(1974, Sec. 3.2), the "Americans":

The subject's choices reveal the existence of: 1) a 'subjective probability measure', $\bar{p}$, on the set $(1,\ldots,n)$ of events and 2) a real-valued function of the 'consequences' q_i $(i=1,\ldots,n)$, denoted by

$\bar{s}(q_i)$, for 'satisfaction'	$B(q_i)$, for 'Bernoullian utility'

with the following property: when offered to choose from a set whose generic element is a 'prospect' of receiving q_i ($i=1,\ldots,n$) when i happens, the subject will choose a prospect which maximizes

a function of M and Σ^2 increasing in M, where $M \equiv \Sigma \bar{p}_i \, \bar{s}(q_i)$ $\Sigma^2 \equiv \Sigma \bar{p}_i (\bar{s}(q_i) - M)^2$ [≡ 'expectation and dispersion of psychological values'; notations in Allais, Note (33)]	M_B, where $M_B \equiv \Sigma \bar{p}_i \, B(q_i)$ (≡ 'expected utility').

II.2 Discreteness of events. As in much of Allais' article the case when the set of events is continuous has been omitted on Figure 1, for simplicity's sake.

II.3 Symbol $\bar{p}$. For subjective probability, we have used the same symbol $\bar{p}$ in stating both hypotheses. For the present author and, I believe, others of the "American School" do agree with the suggestion (Allais, 6) to elicit $\bar{p}_i$ by Emile Borel's method: that is, to observe the subject's choices between a bet on the event i and a succession of bets on drawing a red ball from a bag with an appropriately varying proportion of red balls, known each time to the subject. It must be added, in the 'subjectivist' spirit, that preliminary tests should assure the experimenter beforehand that the subject is indifferent between bets on any of the individual balls, thus revealing that he is indeed assigning to

each the same subjective probability of being drawn (= 1/100 if there are 100 balls).[3)]

II.4 Symbols $\bar{\underline{s}}$ and B. By contrast, the symbols $\bar{\underline{s}}(\underline{q}_i)$ and $\underline{B}(\underline{q}_i)$ denote, in general, different quantities, to be revealed, in principle, by different experiments. The function $\bar{\underline{s}}$ is to be "determined by introspective observation of equivalent increments or minimum perceptual thresholds" [Allais: Summary, 12°; also text, 11; and note (21).] The function $\underline{B}$, on the other hand, is to be determined by observing the subject's choices between bets: see, e.g., Mosteller and Nogee, cited by Allais; Swalm (1966) applied the method to study the choice behavior of business executives.

II.5 Real-valued and other consequences. In the bulk of Allais' discussion the consequences $\underline{q}_i$ are identified with monetary gains and thus can be represented by (possibly negative) real numbers. He does, however, refer occasionally (Allais, 7) to a preference function on the set of vectors whose components are real numbers, viz., the quantities consumed. One might, then, consider as a "prospect" a probability distribution of such vectors: points in a multi-dimensional real space. Under the name of 'multi-attribute choices under uncertainty' (see, e.g., Raiffa, 1968, Chapter 9) the problem has recently acquired great practical importance in applications to public policy.

Still more generally, the consequences need not be numerical, of any dimensionality. The set of consequences $(\underline{q}_1, \underline{q}_2, \ldots)$ may be, for example: 'victory, defeat, stalemate'; 'good health,

pneumonia, tuberculosis'; and so on. This extension of the choice problem to non-numerical consequences -- thus bringing political, military, medical decisions to join economic ones -- is one of the greatest merits of von Neumann and Morgenstern. It has been widely applied by the "American School"; and nothing stands in the way of thus generalizing also Allais' hypothesis.

II.6 Effect of the shape of the satisfaction function $\bar{s}$ on the desirability of the variance of money gains. If the consequences g_i are real-valued (money gains or, for that matter, the number of patients cured or prisoners taken), expand $\bar{s}(g_i)$ in the neighborhood of some arbitrary origin:

$$\bar{s}(g_i)=\bar{s}(0)+g_i\cdot\bar{s}'(0)+g_i^2\cdot\bar{s}''(0)/2+g_i^3\cdot\bar{s}'''(0)/6+\dots ;$$

then the "expectation of psychological values" is

$$M = \Sigma\bar{p}_i\bar{s}(g_i)=$$

$$=\bar{s}(0)+\mu_1\bar{s}'(0)+\mu_2\cdot\bar{s}''(0)/2+\mu_3\cdot\bar{s}'''(0)/6+\dots,$$

where the μ_k (k=1,2,...) are the moments of the distribution $\bar{p}$ about the origin. If the second derivative $\bar{s}''$ is negative (positive) over the considered range of money gains, the function $\bar{s}$ is strictly concave (strictly convex) over that range, and we have the case of diminishing (increasing) 'marginal satisfaction': Allais, 14. Accordingly, when all moments but μ_2 are fixed, M, the expectation of psychological values, will decrease (increase)

when the variance ($\mu_2 - \mu_1^2$) of the distribution $\underline{p}$ increases. To explain why a subject, over a given range of monetary gains, prefers their lower (higher) variance it is thus sufficient to assume that he maximizes $\underline{M}$ and has a concave (convex) satisfaction function $\underline{\bar{s}}$ that, along with $\underline{\bar{p}}$, describes his 'psychology'. It is not necessary to assume that, in addition, it must be characterized by his preferring, for $\underline{M}$ given, a low or a high 'dispersion of psychological values', Σ^2. This is clearly stated by Allais in the first two paragraphs of his section 21. But the clarity is removed in his subsequent paragraphs.

II.7 Analogous effect of the shape of the utility function $\underline{B}$. In the above expansions, replace $\underline{\bar{s}}$ by $\underline{B}$ and, correspondingly, 'marginal satisfaction' by 'marginal utility'. This reconstitutes, in part, Alfred Marshall's (1890, Mathematical Appendix 9) reasoning: if the Bernoullian decision-maker has decreasing marginal utility of money he will prefer lower to higher variance of money gains. More particularly, Marshall seems to have regarded a strictly concave money utility function $\underline{B}$ as a psychological fact -- or perhaps as a moral prescription against gambling, in a Victorian spirit, a heir to Calvin (Allais, Note (52))?.. More generally, M. Friedman and L.J. Savage, in a paper cited by Allais, explained the gambling of people who are insured, by the Bernoullian function being convex in some neighborhoods and concave in others...The same reasoning might apply to Allais' satisfaction function $\underline{\bar{s}}$.

Note the advantage of restricting a hypothesis as little

as possible. Maximizing moral expectation is less restrictive than maximizing expected gain. Concave utility is less restrictive than Bernoulli's logarithmic one. And the inflexions of the money utility curve, introduced by Friedman and Savage are less restrictive and more realistic still. (On the other hand some considerations may impose restrictions: as pointed out by Arrow the utility function must be bounded, to meet the Petersburg paradox!)

II.8 Predicting choices from words or from observed choices? In Borel's method recommended by Allais and by ourselves, the subject's choices are observed. This is also the case when students of Bernoullian utility try to elicit it, -- sometimes by actually paying the subject's lottery gains. On the other hand, the "introspective" comparison of "satisfaction increments" by the subject provides the experimenter with words not with observed choices; so would the subject's naming probability numbers, or their increments. I suppose we are more interested in predicting actions than words; and such predictions are probably better based on recorded actions than on recorded words.

II.9 If one could assume that, by good luck, the functions $\underline{S}$ and $\underline{B}$ do coincide, it would be possible to test the "American" hypothesis against Allais' in the special form of his note (33), equation (8): the decision-maker maximizes

$$\underline{M} - \lambda \Sigma^2$$

[but see also Allais, note (85)]. Consider a triple of probability distributions ψ_1, ψ_2, ψ_3 below; M and Σ^2 are evaluated

in the last two columns:

	$s(g) = 0$	1	4	M	Σ^2
ψ_1	0	.5	.5	2.5	2.25
ψ_2	.25	.5	.25	1.5	2.25
ψ_3	.5	.5	0	.5	.25

If the functions $\underline{s}$ and $\underline{B}$ are identical, so are the quantities $\underline{M}$ and $\underline{M}_B$; and the "American" hypothesis would predict the ordering $\psi_1 > \psi_2 > \psi_3$ of prospects. But Allais' hypothesis would predict this ordering only if $\lambda > 1/2$. It would predict $\psi_3 > \psi_1 > \psi_2$ if $\lambda > 5/4$, and $\psi_1 \geq \psi_3 \geq \psi_2$ in the intermediate case. For a consistent subject a series of such experiments would decide the issue.

III. ACTUAL BEHAVIOR

Common to both hypotheses of our Figure 1 are certain underlying assumptions about human behavior. Unfortunately, they are often contradicted by facts.

Are we sure that -- as assumed by Allais, 4 -- people do derive the subjective probabilities of compound prospects by properly combining those of the constituent prospects? This is made very doubtful by the experiments of Tversky and Kahneman (1974), and the studies of Slovic, Kunreuther and White (1974). It would be worthwhile to follow them up, using subjective probabilities established by Borel's method.

Further: the assumption that a person ranks the prospects in order of preference -- Allais, 2; Savage's Postulate 1 --

was violated in many experiments: by 27% of students tested by May (1954), by 10% of students tested by Davidson and Marschak (1959); although, interestingly, by only 4% of business executives tested by MacCrimmon (1965). Experimental psychologists (unlike the economists and some logicians) are not astonished. It is not even sure that a person consistently chooses the same object from an offered set, even within the same hour or day. As in the experiments of Fechner (1860) on the perceived comparative heaviness or loudness, to the sentence "the person chooses this particular object" the words "with such and such probability" are added. Accordingly, preference ordering is replaced by various competing "stochastic models of choice". The transitivity requirement appears in modified, weakened forms. One of these implies the existence of a numerical function $\underline{f}$ on the set of objects with the following property: the probability of choosing $\underline{A}$ rather than $\underline{B}$ is the larger, the larger the difference $\underline{f}(A) - \underline{f}(B)$. Thus $\underline{f}(A)$ is analogous to Fechner's measure of subjective sensation of the stimulus $\underline{A}$. It may be called 'stochastic utility' or, for that matter, 'stochastic satisfaction'[4]. Other models, both weaker and stronger have been proposed and some of them were submitted to tests[5].

In our Section I we described how a subject may be led to reflect; and to regret his failure to rank objects consistently. Similarly with what Allais, 3, calls the 'axiom of absolute preferences' (also known as the 'admissibility requirement of A. Wald (1940), and identical with Savage's "Theorem 3.")

Here is a test (see Marschak (1968), Section III). Let the subject pick up one of many sealed envelopes, with the experimenter not present. The subject opens it, it contains $\underline{v}$ dollars. He tells a third person his asking price, $\underline{a}$ dollars, for the contents of the envelope, after being told that there is a written bid, $\underline{x}$ dollars; and that if the bid exceeds his asking price he will receive $\underline{x}$ dollars, and will otherwise keep the $\underline{v}$ dollars.

If the subject would tabulate the payoff matrix showing the results of the bid $\underline{x}$ when he has asked $\underline{a}$ dollars, he would see that he should ask the amount $\underline{a}=\underline{v}$. A lower or higher price asked will not give him, for any level of $\underline{x}$, any higher results and will, for some ranges of $\underline{x}$, give him less. In my experience with students and in K. MacCrimmon's with business executives, not using pencil and paper, they usually ask more than $\underline{v}$, thus violating the principle of absolute preferences. After being invited to tabulate and to reflect they 'regretted'.

Most psychological studies of children, but also of adults, the bulk of the anthropology of primitive people, much of sociology and, of course, all of psychiatry deal with actual behavior. But it is also supposed that psychiatrists, and psychologically trained teachers, will help the patient or the pupil to approach in their behavior some norms of logic, of arithmetic. It is similar with the decision behavior: it is important to describe, but also to improve it.

IV. TRAINING FOR RATIONALITY

IV.1 Young children are taught arithmetic (and, nowadays, also set-theoretical logic) using the abacus, colored blocks etc. and avoiding technical words as long as possible (except for bad teachers and bad writers of textbooks of new maths). Also, to train people not to confuse "if" and "only if", simple exercises must be used.

I believe that, similarly, procedures can be developed to train people for decision-making. To educate future decision-makers, possibly future persons with great responsibilities, is a task similar to, and socially at least as important as, the education of future accountants and lawyers in arithmetic and logic.

Psychologists practiced in cognitive experimentation (studies of problem-solving) know how the manner of presenting a problem, -- even the trivial, detailed features of its presentation, -- affect the subject's ability to get the necessary insight. At the same time as MacCrimmon presented problems to executives, I used the same problems with the students in my class. But by the time I presented the "substitution" problem my students had become used to drawing payoff matrices, thus translating the verbal, syntactically complicated statement of the problem into a visually clear form -- unlike MacCrimmon's subjects. (You may have noticed, when discussing a car purchase with a dealer, that he uses the paper for doodling, placing figures, percentage calculations and additions in odd corners of the sheet and foregoes the advantages of clear visual sequences or tables). A large majority of my students obeyed

the substitution axiom, in contrast to MacCrimmon's executives, and this may be due to their having drawn up and contemplated payoff matrices.

As quoted in Section I above, Savage did change his decision, and complied with his axiom, after having drawn payoff matrices. In case of a less sophisticated person, trainer for rationality would also succeed, I believe, if Allais' experiment on the substitution postulate were not phrased in terms of numerical probabilities, or numbers of lottery tickets, but in terms of events: to-morrow's sunshine, clouds, rain, gale.. This would avoid the influence of any school-bench memories of urns and chances, yet capture the essence of the experiment, and of its possible application in real life. (Remember also, that Savage's subjective probabilities do not exist for a person disobeying his axioms!)

While Savage, on reflection, changed his decision, "nobody could convince the author (viz., Allais, 71) to change his mind should he <u>as a practical matter</u> find himself in the happy position of facing the uncertain prospects described" (i.e. with a small chance of an immense gain, yet, in Savage's 'contradicting' Allais previous choice.) This difference in attitudes suggests that it may be worthwhile considering methods of training for alternative types of 'rationality! It should be possible to face a person with decisions that would gradually reveal to him <u>his</u> true satisfaction function $\underline{\bar{s}}$ and the other function also characteristic of his tastes, which, according to

Allais, depends on the expectation and the dispersion of 'psychological values'.

This raises the question of possible modificating of norms. Recently, thoughtful authors (P. Fishburn, (1970), A. Sen, (1969)), while retaining the transitivity of preference, have proposed to discard the transitivity of indifference, essentially on the grounds of non-discriminability of small increments: while I prefer 110 to 100 cents, I am indifferent between 100 and 105 cents, and between 105 and 110; and so is the automatic computer, condemned to rounding-offs. And A. Tversky (1972) has deepened both the logic and the psychology of discrimination as a concomitant of choices. One might say it is indeed non-rational, non-economical to avoid rounding-offs. Both engineers and accountants do round off. Even replacing deterministic by stochastic decision models may be "economical": it requires too much effort to be perfectly consistent!

I suggest then, that

1) the most obvious modifications (mostly weakenings) of 'rational' axioms should be formulated and their logical implications studied. That is, the personal utilities and probabilities would exist only as "approximations" -- this term to be defined as precisely as possible and applied operationally to well-trained decision-makers.

2) Effective, partly non-verbal <u>training devices</u> should be developed which would make a given modified norm to a habit-similar to the mathematician's or a good lawyer habit

of distinguishing between necessary and sufficient conditions.

The task of developing methods of training is a psychological one. It must be distinguished from another, also psychological, or anthropological, task, more in the tradition of these disciplines: to describe the actual behavior of people of a given culture, social group, age, physical condition.. A decision-maker dealing with people will need this information -- as an engineering decision-maker may need information on the properties of metals. At the same time, both must be trained to apply norms of logic, mathematics, and of decision-making, albeit modified ("rounded-off") to avoid uneconomical efforts.

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NOTES

1) The present comments deal with M. Allais' mimeographed paper No. 2824, Centre d'Analyse Economique, which, I understand, is being reproduced in the present volume. I shall cite by number the sections and notes (not the pages) of that paper. As far as possible, the notations of M. Allais will be used. My list of references (provided with the publication year in the text) will include only those not already contained in the bibliography attached to M. Allais' paper.

2) "Two very different sorts of interpretations can be made of... the ... postulates.. First, ... as a prediction about the behavior of people, or animals, in decision situations. Second, ... as a logic-like interpretation of consistency in decision situations. For us, the second interpretation is the only one of direct relevance, but it may be fruitful to discuss both, calling the first *empirical* and the second *normative*". I strongly recommend pondering Savage's subsequent discussion of the two interpretations. (*Foundations*, 2.6).

3) I take this opportunity to correct M. Allais' impression (6, case V) that I have persisted in the 'objectivist' stance of my early articles of 1950-51. On the contrary: see Marschak (1954); (1954a); (1968); (1970). Borel's procedure is described in the latest two of these essays. This and other methods of eliciting subjective probabilities were surveyed by Savage in his posthumous paper (1971).

4) Fechner tells us that when assuming the 'sensation' to be

logarithmic in the amount of the physical stimulus he was inspired by Daniel Bernoulli's logarithmic function (our $\underline{B}$) of the gambler's achieved amount of money. It is only fair that students of gambling now, in return, imitate Fechner's stochastic hypothesis!

5) Luce's pioneering work (1959) was followed by various authors including my collaborators and myself: (1959, 1960, 1960a, 1963, 1963a, 1963b, 1964). For a survey see Luce and Suppes (1965).

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